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ABSTRACT
The model described is designed to provide the educational diagnostician with data relative to an individual's content deficiencies in mathematics, mathematics cognitive style, and educational cognitive style. A diagnosis of these three factors requires consideration of cognitive, affective, and psychomotor concerns. The diagnostic mapping of an individual includes personality factors as well as mathematical strengths and weaknesses. This mapping is used in conjunction with a mapping of available instructional resources in order to prescribe an effective remedial mathematics procedure. (Author/SD)

A Clinical Model for Diagnosing Mathematical Deficiencies, $(MD)^2$ Incorporating Educational Cognitive Style

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Introduction

One of the most significant movements toward improved instruction that has been made in recent years is that of incorporating into the instructional setting plans for diagnosing learning difficulties and detecting needs for preventive and corrective teaching (Engelhardt, 1974, 1976; Irons, 1976). In fact, the term "diagnosis" and its variety of meanings have become part of the vocabulary of mathematics educators, with some notable exceptions, at all levels of instruction (Crenson, 1976). As an example, consider the Guidelines for the Preparation of Teachers of Mathematics (1973) published by the Commission on Preservice Education of Teachers of Mathematics of the National Council of Teachers of Mathematics which put forth recommendations that included the attainment of competencies in "recognizing stages of cognitive, affective, and psychomotor development in children and individual differences between children as these differences pertain to the learning of mathematics," (p. 15) as well as in diagnosing and prescribing "remedies for common disabilities in the learning of mathematics and to know what tools and techniques are available to help with diagnosis and correction" (p. 15).

Nevertheless, when the literature dealing with the diagnosis of mathematical difficulties is examined, one finds "a paucity of relevant research and a lack of substantive contributions" (Cawley, 1975, p. 12). "Research, in the main, has been perfunctory at best and dissemination almost non-existent" (Crenson, 1976, p. 1). Smiler (1970), in the National Council of Teachers of Mathematics Thirty-Second Yearbook, reported a study which categorized research studies in mathematics education made in the United States between 1880 and 1963. The category focusing on diagnosis contained a total of only 103 entries, which in graphical form, reflects a bimodal distribution with the modes appearing in the late 1920's and again in the early 1940's. A similar study by Suydam (1970) located a total of 107 studies focusing on diagnosis and/or remediation for the period ranging from 1900 to 1965. Wilson, reporting on the nature of research in mathematics education, stated that any search of available resources

> readily reveals that the type of work I've called clinical intervention [guiding efforts in diagnosis

> and treatment of children's learning difficulties in mathematics/ is either not recognized as research, or is depreciated as research, and in some circles even deprecated as research. (Wilson, 1973, pp. 1-2)

Several recent calls for needed research appear to be having an impact with respect to generating new interest in research and application of diagnostic models (Gray, 1972; National Advisory Committee on Mathematical Education, 1975; National Assessment of Educational Progress, 1973; National Conference on Needed Research in Mathematics Education, 1967; National Conference on Remedial Mathematics, 1974, 1975, 1976). A cursory examination of recent National Council of Teachers of Mathematics regional and national meetings indicates expanding attention to the diagnostic-remedial arena. State governments also provide impetus for further gains by the passage of laws such as Section 230.2311 of the Florida Statutes (1975) which states, in part, that each school district's program shall include an individualized diagnostic approach to instruction.

> Probably the '70s will witness a kindling of efforts by a national cross-sectional group of individuals from diverse institutions representing isolated efforts in diagnosis in clinics and teacher training programs. The most identifable catalyst in this movement is the effort of Jim Heddens at Kent State University who, through the help of the KEDS General Assistance Center, has organized three national conferences with diagnostic themes. (Underhill, 1976, p. 1)

Callahan in his summary paper of the first National Conference on Remedial Mathematics, stated:

> Many participants commented on the level of knowledge that exists in regard to diagnostic-remedial procedures in mathematics. It would seem fair to say that there is not a great deal of systematic, accumulated knowledge. Some smatherings of research evidence and some sensitive and insightful thoughts on the subject exist. Some isolated individuals at various points in time have attempted to pull together some of the research and thoughts, but the level of scientific knowledge regarding the diagnostic-remedial episode in mathematics is not great. (Callahan, 1974, p. 3)

Many examples have been used to cite this apparent lack of concern for providing a sound knowledge base for research on

diagnosis (National Conference on Needed Research in Mathematics Education, 1967). What is perhaps more important is why this state of affairs exists. Engelhardt (1974) proposed three major factors as possible explanations. These are (1) "an insufficient understanding of the learning process," (2) "the disjointed nature of most research efforts," and (3) "the apparent lack of theoretical models for mathematical remediation, models which would identify sets of variables to provide a common focus for research" (pp. 2-3). These three factors provide the framework for this paper.

Purpose

The purpose of this paper is to describe a theoretical model for the clinical diagnosis of mathematical deficiencies--a model which is compatible with and built upon research and supported by theory. The research base for a study of this nature is not confirmative (i.e., one which is experimental) but rather it is generative (i.e., one which generates hypotheses) and analytic-synthetic (i.e., one which constructs guiding models and explanatory theories) (Wilson, 1973).

Specifically, the model focuses on three considerations, namely, the identification of specific mathematical deficiencies, the identification of mathematical cognitive style, and the identification of general educational cognitive style. The model is designed and developed primarily for use in a clinical setting.

Identification of specific mathematical deficiencies refers to the process of determining, by various forms of examination the specific nature and circumstances of a more general, suspected deficiency. For the purposes of this paper the diagnosis of mathematical deficiencies is limited to the concept clusters represented by the Kent State University Mathematics Checklist (1975). It is important to note, however, that the possibility exists for the extension of this model into other areas and levels of mathematics. Indeed, if this model is to provide any significant contribution to the field of diagnosis it must have this property. "The creative aspects of mathematics, skill in searching out mathematical patterns, non-routine problem solving, and the study of functional relationships must be considered as carefully as the skill aspects" (Riedesel, 1974, p. 1).

Mathematical cognitive style is viewed as the manner in which an individual operates on the concrete--representative--abstract hierarchy in light of mediating variables (i.e., visual, auditory, tactile) encountered in both the mode of presentation and type of desired student response.

Educational cognitive style, as it is used in this paper has its foundation in the work done by Hill (1968, 1972, 1974).

Informally, educational cognitive style is the way in which an individual takes on meaning, the way in which an individual perceives his surroundings, the way in which an individual can master an educational task most readily (Hill, 1974).

The purpose of the paper, then, is to combine the three factors of mathematical diagnosis, mathematical cognitive style, and educational cognitive style into a model and instrument which can be used to generate a map of the individual student. This map can then be used in a prescriptive sense to determine the most probable means of providing successful corrective teaching.

Theoretical Framework

One of the assumptions of this paper is that "man is not content with biological satisfactions alone, but rather he continually seeks meaning" (Hill, 1974, p. 2). The implication is, of course, that we are inherently curious and constantly searching for reasons and explanations which give meaning to our environment as we perceive it. The fruits of our search for knowledge and understanding, in this instance, are represented by a profusion of theories and models which attempt to explain various aspects of the educational arena.

Some theories are formulated on the basis of painstaking observation and categorization--what has been called "confirmative research" (Wilson, 1973). The foundation of such theories lies in the gathering of experimental data which is then used to develop classification schemata. In mathematics, for example, this is akin to the inductive reasoning process used by elementary school children who generalize, after several experiments, that the sum of two even numbers is an even number.

Other theories are predicated on the basis of rational analysis. That is to say, a general theory is put forth, based on certain assumptions, and then "experiments" or observations are made to see whether or not they fit the theory. Essentially, this is the process called "empirical mapping" by Hill (1974), or "generative research" by Wilson (1973). An example of such a process can be found in the works of Piaget (1952).

An important characteristic of confirmative or experimental research has been described by Mouly.

> Experimentation, whether in education or any other field, rests on the assumption that there exist invariant relationships between certain antecedents and certain consequents so that, provided a given set of conditions prevail, if one does <u>this</u>, <u>that</u> will follow. (Mouly, 1968, p. 6).

But, exceptions to the "rule" do exist. In education it is particularly easy to rationalize these exceptions into oblivion by calling on some unusual circumstance as the cause of the unexpected exception. In many instances, of course, there may indeed be some "unusual circumstance" acting. The multitude of variables involved in many educational studies surely increase our chances of overlooking an important factor.

> The difference between what was planned and actually occurred is considerable. Furthermore, this difference is intended. Corrective instructional events are not mechanistic routines to be blindly followed. Real events grow, change and develop as the human beings involved in the event interact. (Romberg, 1976, p. 3)

Nevertheless, this technique of rationalizing identified contradictions sometimes amounts to little more than burying our heads in the sand. About the best that can be said is that experimentation in education has produced a number of generalizations, principles, and laws which are valid under certain stated conditions (Van Dalen, 1966; Romberg, 1976).

Information gathered in the name of rational analysis, personnal experience, intuition, or opinion has the same characteristic. A great deal of the "knowledge" we possess relative to our students has been derived through informal observation, and consequently generalizations, principles, and laws formulated in this manner are valid under certain stated conditions. It appears, at least to the author, that when we deal with the "less tangible," less experimentally oriented aspects of education there is little difference between the experimental and experiential approaches, except perhaps that in the experiential model we do not try to rationalize inconsistencies into oblivion (Nunnally, 1975).

> Instead of making generalizations the ruling consideration in our research, I suggest that we reverse our priorities. An observer collecting data in one particular situation is in a position to appraise a practice or proposition in that setting, observing effects in context. In trying to describe and account for what happened, he will give attention to whatever variables were controlled, but he will give equally careful attention to uncontrolled conditions, to personal characteristics, and to events that occurred during treatment and measurement. As he goes from situation to situation his first task is to describe and

> interpret the effect anew in each locale, perhaps taking into account factors that were unique to that locale or series of events. (Cronbach, 1975, p. 117)

This is essentially what Romberg (1976) called "process evaluation"; "process" in the sense that the focus is directed toward actions as opposed to outcomes, and "evaluation" in the sense that it is a question raising search rather than conclusion drawing research.

Wilson summarized the position of research of this nature in the following manner.

> The reasons for neglecting the systematic development and use of clinical intervention as a type of research are varied and have deep historical roots. Of those which are most often mentioned to me, the most common are that "such studies aren't 'rigorous'", "they don't use controls", "they use procedures and data that don't lend themselves to the analyses of inferential statistics", "you can't generalize from one child or even a small group so studied", "there is no way to replicate", etc.
>
> Such criticisms are based on criteria appropriate to experimental research. For studies that are labeled as experiments and intended by the researcher to fulfill the purposes of experimental research, such criticisms are, of course, accurate and fully justified.
>
> That studies which do not claim to be experiments are also criticized on these grounds--if only implicity--attest to the eminent position the criteria of excellence in experimental research have attained in our community. The high esteem we have for correctly designed and executed experimental research is fully justified--for the purposes to which experimental research is suited. A somewhat comparable and justified esteem is held for sound correlational studies. But is it possible this esteem has obscured our clear recognition of the potential value of other kinds of research? In turn has this inhibited our efforts to improve other kinds of research? (Wilson, 1973, p. 2)

One often hears, particularly in educational contexts, that theory and practice are not even related, let alone isomorphic

(Newsome, 1964). Others, including the author, do not agree with this position. Reys and Post (1973) stated that, "Facts play a central role in the development of theory, and the theory subsequently provides a systematic interpretation of the general area to which the facts are related" (p. 16). Hill stated that

> It is a serious mistake to think of a realm of theory that is separate and different from the realm of fact. It would be reasonable to say either that facts represent one kind of theory or that theories represent one kind of fact, but most reasonable to say that fact and theory represent different degrees of what is basically a single process. (Hill, 1963, p. 23)

Burns (1962) suggested that the nature of the relation between theory and practice can best be described by "pragmatic implication" which he defines as "a rational person with certain beliefs relevant to certain kinds of situations generally acts in accord with those beliefs" (p. 54). Guttchen (1966) took exception to Burns' approach, mainly because the terms "rational" and "relevant" seem to leave too much room for different interpretation. He did, however, allow for the possibilities of action according to pragmatic implication in such areas as medicine or engineering.

Gowin (1963) posed the view that theory is a blending of logic and facts and that it is best seen as a guide to thought and inquiry. In other words, Gowin points to the true nature of the relationship between the strategies of studies done in an experimental vein and those founded on rational analysis. Perkinson (1964) preferred to couch educational theory in terms of a strategy--what Gowin (1964) referred to as a flow chart to guide experimentation.

Clements (1962) described two basic types of theories, namely "prescriptive" and "descriptive." Of these two, the former represents what is generally called "educational theory." According to Soltis (1968), "Descriptive theories are adequate when they allow for accurate predictions and little, if any, educational theory is now of this sort" (p. 85).

Two opposing views are presented by Reys and Post (1973) and Newsome (1964). Reys and Post contended that, "Ideally, theories should provide insight to both theorist and practitioner concerned with a common area of investigation" (p. 17). Newsome, on the other hand, argued that the relationship between theory and practice is negligible. Theory does allow for better understanding of practical situations, according to Newsome, but it does not describe a set of logical processes to be applied to any given situation.

Whatever the organizing foundation for theory may be, the development of the general principles and laws contained within the structure of the theory provide us with the means to at least attempt to predict and control events in our surroundings. The degree to which we have predictability and control is dependent upon several factors including "goodness of fit (how much agreement there is between the model and the phenomena it is attempting to describe), "relevance" (the degree to which the theory matches other characterizations, particularly those that have "checked out") and "fruitfulness" (the development of checkable characterizations beyond those already in existence).

Golladay, DeVault, Fox, and Skuldt (1975) identified problems in empirical research in mathematics education. It is argued that it is often difficult to choose "appropriate conceptualizations and measures for a variety of phenomena" (p. 159) found in the study of individuals and individual differences. Further, "models of more traditional, structured educational experiences are not appropriate for examining the greater variety of opportunities and experiences characteristic of most individualized programs" (p. 160). A call is made for the use of paradigms, i.e., descriptors, to identify categories and relations with the intent of organizing observed data. They pointed to the successful use, by the scientific community, of paradigms but quickly draw from a study by Apple (1973) which indicated that while educators often employ paradigms they are seldom specifically stated.

Golladay et al. also pointed to the problem of reliability in studies which focus on the individual. The major cause is the complexity of events which are presented to the observer. They concluded that

> It may well be inappropriate to search for traditional methods for testing the reliability of information when the program being studied departs from traditional patterns ... and information is gathered in a manner different from that of traditional research designs. (Golladay et al., 1975, p. 168)

Since the present paper is concerned with the development of a model for use in diagnosing an individual's mathematical deficiencies and identifying individual educational cognitive style it seems that the notions presented in the previous pages, are relevant. However, an additional word of caution is necessary. An inherent danger of developing an illustrative model (or theory, for that matter) is oversimplification to the point that distortion makes the model (or theory) useless. On the other hand, presenting a model (or theory)

which incorporates all of the complexities of the situation under study runs the danger of being too copious to allow for practical application.

Nature of Diagnostic Models

Diagnostic models are of three varieties in terms of the setting in which the diagnosis is to take place, namely, models that are classroom oriented, models designed for clinical use, and models which may be applied to either environment. The primary emphasis in this paper is on clinical models since the $(MD)^2$ model is of this type.

Diagnostic models can also be differentiated on the basis of their assumptions concerning the purpose of diagnosis and its corresponding methodologies.

One type of model, the ability training model, has as its purpose the identification of learner capabilities which, when identified, may be used to prescribe corrective teaching (Uprichard, Baker, Dinkel & Archer, 1975). Thus, ability training may be roughly equated with aptitude-treatment-interaction (ATI) which "seeks to provide a basis for employing differential treatments in order to exploit the cognitive preferences displayed by different individuals for differing content or mode of instruction" (Hancock, 1975, p. 37).

Some objections to the use of the ability training model that have been cited include: (1) the nature of the operational definitions used, (2) the difficulty of incorporating ATI findings in the instructional setting, (3) the instability of reliability and validity measures of instruments used to gather data, and (4) the lack of research which supports the notion that remediating weaknesses in cognitive preferences leads to increased performance in the classroom (Stiglmeier, 1972; Uprichard et al., 1975).

Each of these objections can be countered by referring to the available literature. For example, objections one and two can be dispelled by providing a scientific framework for education and the accompanying means of implementing this framework in educational settings (Hill, 1974). Objection three has been discussed at great length (Hill, 1973) and poses no problem provided results are properly interpreted. Concerning objection four, it has been stated that, "the lack of productivity in this area has been ascribed to inadequacies in research design and general methodology" (Cunningham, 1975, p. 171). However, there now exists an abundance of research which refutes the objection that application of ability training techniques does not increase achievement. Not only has it been shown that remediating weakness in the child's cognitive preferences leads to increased performance in learning situations but also it has been

shown that remediation based on utilizing the child's cognitive preference strengths had similar effects (Radike, 1973).

The second theoretical model for diagnosis is known as the task-analysis model. This content oriented method consists of "analyzing a learning task into a hierarchy of subordinate tasks, diagnosing the pupils' mastery of the subordinate tasks and giving instruction in the specific subordinate tasks not mastered by the learner" (Callahan & Robinson, 1973, p. 579). According to Uprichard et al. (1975), in task analysis "the emphasis is on component skills and their integration into complex terminal tasks rather than the processes that presumably underlie the development of specific tasks" (p. 2).

Identified criticisms of this model include: (1) the content-orientation may cause the diagnostician to overlook important factors in the student, (2) the task analysis of certain subjects is difficult, (3) the validation of hierarchies is a difficult process, and (4) the prescriptive philosophy of task analysis tends to be founded on changes in the curriculum, for the most part ignoring changes which reflect analysis of the learners' cognitive style (Uprichard et al., 1975).

Despite these criticisms several studies have pointed to the value of the task analysis model. For example, Uprichard et al. (1975) stated that "the task analysis model has appeal for /mathematics/ educators since the structure of the discipline aids in the building of hierarchical relationships.... /it/ is valuable in that the diagnostic findings rely on fewer undefined and unvalidated assumptions" (p. 2). Additional benefits of the task analysis model have been suggested.

> One conjecture is that a procedure of diagnosis and instruction based on a hierarchical analysis or subordinate tasks is an effective procedure for students' learning of a mathematical task.... Another conjecture is that where the task analysis procedure is used in the teaching of a mathematical task the incidence of underachievement ... will significantly decrease.... In summary, the task-analysis procedure when combined with meaningful mastery learning of the subordinate tasks in a hierarchy seems quite effective in learning a mathematical task. (Callahan & Robinson, 1973, pp. 583-584)

It should be noted at this point that the model described in this paper is a synthesis of both the ability training and the task analysis theories of diagnosis. The Model for Diagnosing Mathematical Deficiencies is designed to provide information about student style

as well as content deficiencies. The corrective teaching procedures suggested by the model include not only revisions in content but also in modes of instruction to better match the individual's unique cognitive preferences.

Another method of differentiating between diagnostic models is by identifying their research base, specifically, by referring to the manner in which the various theories of diagnosis are developed.

Wilson (1973) described three major classes of research. The first, confirmative research, is experimental in nature and centers on "activities designed to assess the truth of probable hypotheses" (p. 11). The second, analytic-synthetic research, deals with "activities involved in the development of guiding paradigms and explanatory theories" (p. 11). Third, generative research, is based on "activities consistent with the postulates of science designed to generate hypotheses with an a priori probability" (p. 10). Generative research is further described by Wilson in terms of two subtheories: (1) normative research with "activities designed to generate hypotheses concerning facts and those connections between facts which exist in nature" (p. 16). and (2) clinical intervention research which involves "activities designed to generate hypotheses on those connections between facts which might be brought into nature by some intervention" (p. 16).

The model described in the present paper, the clinical Model for Diagnosing Mathematical Deficiencies--$(MD)^2$, has a research base which is both generative and analytic-synthetic. Aptitude-treatment-interaction theories are also founded on the generative research approach by nature of their study of the relevant processes engaged in by students in learning situations with the intent of generating hypotheses concerning these processes (Wittrock, 1974). Much of Piaget's work has centered on this same form of research. Piagetian-type research has impelled us to

> take a fresh look at our field and to ask a host of new questions concerning the nature of developmental stages and of developmental processes generally, as well as of the kind of research approaches which the study of these problems demands. In so doing it has helped us appreciate the important place of systematic theory in an area of developmental research, essentially comparative in nature, which has not always been noted for its theoretical sophistication. On the other hand, the theoretical significance of research inspired by Piaget's ideas does not prevent it from having direct and important relevance

> for the resolution of practical questions of pedagogy and educational practice. (Wohlwill, 1968, p. 446)

To summarize, diagnostic models can be differentiated in terms of whether they focus on classroom and/or clinical procedures, on whether they are ability-training or task analysis oriented, and on whether they have a research base which is confirmative, generative, or analytic-synthetic. These different approaches to the diagnostic-prescriptive arena do not necessarily reflect differences on what diagnosis is, but rather on how diagnosis is to be carried out.

Nature of Educational Cognitive Style

Before discussing the $(MD)^2$ model, it is necessary to examine one of its components in some detail. This component, educational cognitive style, is not content oriented toward mathematics but its usefulness in the prescriptive stage will readily become evident.

> There will be a great change made in the first and foremost and continuing business of society: the education and training of the young. The development of the mind of the child will come to rest in the knowledge and skills of the biochemist, the pharmacologist, and neurologist, and psychologist, and educator. And there will be a new expert abroad in the land--the psychoneurobiochemeducator. (Krech, 1969, p. 374)

While the "new expert" that Krech refers to may still be somewhat futuristic, advances have been made to develop a more scientific, but not less humanistic, framework for education. The most notable of these efforts falls under the auspices of the American Educational Sciences Association and its founder, Joseph Hill. Since its inception in 1971 (Hill's first published work in this area was in 1966) its membership has grown to over 250 (AESA Membership Directory, 1975) and a recent AESA bibliography (Berry, Sutton, & McBeth, 1975) included over 300 entries on various aspects of the Educational Sciences. Also, the educational science of cognitive style has had considerable impact on educational programs at all levels and has been adopted by many school systems. It is recognized that shear numbers are no indication of the value of any organization or cause. This data is provided only for the reason that it dispels any thought that knowledge of, and development in, the Educational Sciences is limited to a select few.

The following quotes suggest the rationale for the development of the Educational Sciences.

> American education presents to the public view the spectacle of a house divided against itself. One needs only to persue the back copies of educational journals to see how the battle has raged, and to observe the disarray of the schools. Each conflicting point of view finds its advocates.
>
> It is obvious that the confusion and disarray in education arises from the lack of commonly agreed upon goals, practices, and definitions. In other words, instead of having a common framework and a common language, educators have developed an amorphous collection of ideas, concepts and methods from a variety of other disciplines. (Radike, 1973, Introduction)
>
> Without a framework of 'language', the vast field of human activity called 'education' does not readily lend itself to meaningful description or definition. At the present time, the universe of discourse associated with education lacks precision beyond that found at the levels of common sense and daily journalism. The difficulty with such language is not that it fails to provide a form of communication, but that the possibilities of misunderstanding are great and the probability of relatively precise discriminations and predictions is small. (Hill, 1968, p. 1)
>
> Many educational terms do not have clearly assigned and commonly understood meanings, when words such as 'democracy', 'education', 'curriculum', and 'discipline' are used by different workers in the field, they may stand for slightly or radically different things. In contrast, the technical terms in the exact sciences such as meter, ampere, lightyear, and calorie are instruments of great exactitude. (Van Dalen, 1966, p. 200)

These exact sciences referred to by Van Dalen can be equated to what Hill refers to as "fundamental disciplines" (Hill, 1974).

> Fundamental disciplines are bodies of knowledge generated by communities of scholars that produce pure and distinctive forms of information about phenomena which they study. Biology, history, art, psychology, and mathematics are examples of fundamental disciplines.

> Complementing the fundamental disciplines are the applied or derivative fields of knowledge. These bodies of information are generated by practioners who deal with practical considerations of the human condition. Medicine, pharmacy, engineering, and law are examples of applied fields of knowledge. (Hill, 1974, p. 1)

Education is not a fundamental discipline but instead is an applied or derivative field. The Educational Sciences represent an attempt to describe a conceptual framework for education that is as precise and definite as that found in other applied fields. According to Hill,

> With the development of the Educational Sciences, the solutions of problems and explanations of phenomena are facilitated, and educational problems accuring to inadequate communication, misinterpretation of information, and fragmentation of effort are alleviated. (Hill, 1974, p. 1)

Presently there are seven educational sciences. These include: (1) symbols and their meanings, (2) cultural determinants of the meanings of symbols, (3) modalities of inference, (4) biochemical and electrophysiological aspects of memory, (5) cognitive style of individuals, (6) teaching styles, administrative styles and counseling styles, and (7) systemic analysis decision-making. The fifth educational science, cognitive style of individuals, includes the first three educational sciences (the fourth is not sufficiently developed at this point). Therefore, for the purposes of this paper the discussion will center on the educational sciences of symbols and their meanings, cultural determinants, and modalities of inference, i.e., educational cognitive style.

Classroom teachers have long been aware that students come to know what they know in their own unique way. Until recently, however, there was no established framework for teachers to analyze the learning habits of their students and match them with the "most appropriate" mode of instruction. Educational cognitive style provides such a framework.

Briefly, educational cognitive style is a means of identifying the ways in which an individual perceives and reacts to the environment. An individuals' cognitive style is the way a student tends to seek meaning and the manner in which information is personally filtered. Cognitive styles are influenced by the ways in which individuals derive meaning from symbols related to their personal experiences and the world about them; the influences of family, friends, and their own individuality on these meanings; and the kind of reasoning processes used to derive these meanings.

PAGES 15 and 16 "A BRIEF GUIDE TO COGNITIVE STYLE MAPPING SYMBOLS AND THEIR MEANINGS" REMOVED DUE TO COPYRIGHT RESTRICTIONS

By utilizing the techniques of observation, interview, and preference testing, a diagnostician can gather and assemble data on the elements of each of the three major components of educational cognitive style to form a profile, or "cognitive style map" of the individual student. These elements may appear as major orientations (if the element score occurs in the 50th-90th percentile of a distribution of scores for that element), minor orientations, denoted by a prime (if the element score occurs in the 26th-49th percentile), or negligible orientations (if the element score is at or below the 25th percentile).

The testing procedure used to arrive at a cognitive style map has received considerable discussion elsewhere (Radike, 1973), and consequently a further detailed treatment seems inappropriate. Suffice it to say that the diagnostician may use any one, or all, of the three methods: (1) observation, (2) interview, and (3) preference testing.

> The mapping of cognitive styles is mainly <u>empirical</u> in nature, and as such, is dependent upon the judgments of persons (diagnosticians) ... The cognitive style of an individual cannot be empirically mapped without considering: (1) the level of educational development of the person, (2) the general symbolic conditions of educational tasks he will be called upon to accomplish, (3) certain antecedents (e.g., family) to his present state of development, and (4) the appropriateness of the elements under consideration for the conditions under which the educational tasks must be completed. (Hill, 1970, p. 7)

For those readers that are interested, Radike (1973) presents a valuable summary of the process of empirical mapping.

The educational cognitive style model is similar in some respects to the Task-Process Integration Model (Uprichard et al., 1975) however, it is content-free and considerably more global in its approach to student's learning style. Educational cognitive style diagnosing can best be described as a combination of classroom and clinical procedures and is clearly an ability training model although once the learner diagnosis is complete a form of task-analysis is used in determining the symbolic orientation of instructional resources.

The Model for Diagnosing Mathematical Deficiencies--$(MD)^2$

There has been considerable research focusing on the traits of successful mathematics students (Shuert, 1970). The results, while tending to be inconclusive, do suggest that several factors need to be given greater attention than they may have been given in the past. A partial list of "identified" traits, summarized from Shuert (1970), is found below. [Note that the traits are grouped, roughly, as they relate to symbolic orientations, cultural determinants, and modalities of inference as found in educational cognitive style (Hill, 1974).]

The successful mathematics student

has high general and reflective intelligence
prefers objective, non-personal symbolism
is high in verbal ability and comprehension
is highly competitive
possesses authoritarian attitudes
tends to be insecure and sensitive
tends to avoid social and interpersonal issues
rates high on self-acceptance
is anxious
is concerned with "abstract" beliefs

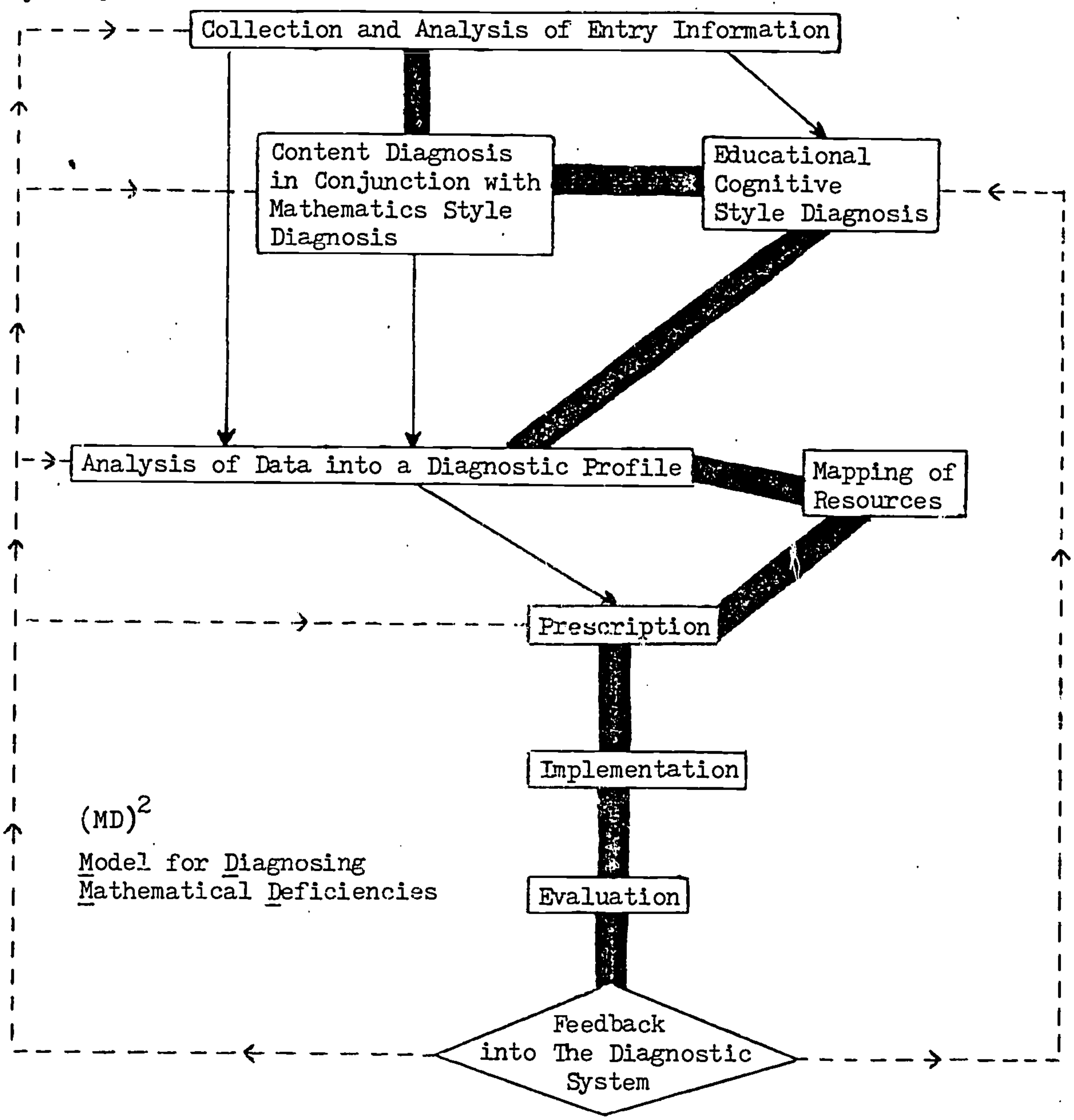
clings to convictions
rejects imposed standards of behavior
prefers to act individually

utilizes both analytical and intuitive processes
finds, organizes, and evaluates relations
has a facility for syllogistic reasoning

The abundance of factors that must be considered in attempting to provide each student with a successful mathematics experience point to the need for a comprehensive diagnostic model at the clinical level (Underhill, 1976).

Some of the questions which must be attended to in the development of such a model include: (1) what general information should be sought, (2) what mathematics content should be considered, (3) what sequence of concepts should be used, (4) what levels of abstraction should be checked, (5) what sensory inputs should be examined, (6) what consideration should be given to affective aspects, (7) what consideration should be given to psycho-motor aspects, and (8) what type of prescriptions should be available. These, and other related questions, form the focusing point for the development of the $(MD)^2$ clinical diagnostic model--A Model for Diagnosing Mathematical Deficiencies.

The model is a three-fold model of diagnosis in that the profile generated for an individual includes data relative to that individual's unique mathematical deficiencies, "mathematics style," and educational cognitive style. This information is gathered and utilized during the course of an intensive analysis of a student using case study techniques. The nine step process involves: (1) collection and analysis of entry information, (2) content diagnosis in conjunction with "mathematics style" diagnosis, (3) educational cognitive style diagnosis, (4) analysis of data from steps 1, 2, and 3 into a diagnostic profile, (5) mapping of resources, (6) prescription, (7) implementation, (8) evaluation, and, if necessary (9) feedback into the diagnostic system.



The first step, the collection and analysis of entry information, is designed to provide general background data concerning the student. Some factors which are included here are parental information, school records, behavior patterns, interests, and anecdotal information such as expressive ability, motivation, self-confidence, attentiveness, and attitude toward mathematics. Every effort should be made to secure reliable data from the student's classroom teacher, school officials, and parents. However, the diagnostician should also engage in the observation of the student with an eye not only toward the analysis of mathematical abilities but also toward those behaviors reflecting the child's physical, psychological, affective, and social orientations. Through an initial interview, as well as other means, the diagnostician should seek to identify such factors as interest, cooperative effort, persistence, flexibility, and adjustment to interview situations. The analysis of the student's reaction to successes, failures, positive reinforcement and negative reinforcement may also provide clues to underlying content deficiencies.

Referral to the $(MD)^2$ model implies that the instigator of the referral has identified some general mathematics content deficiencies or that the instigator simply wants some particular content area diagnosed. These general content areas must be defined and recorded before the $(MD)^2$ model can be implemented. One aspect of this defining process is the interviewing of teachers and parents focusing on their interpretation of what content should be diagnosed. This should then be followed up by an analysis of the student's standardized test results. If no such results are available the diagnostician may request or conduct a standardized diagnostic test such as the Buswell-John Diagnostic Test, KeyMath, or the Stanford Diagnostic Test. This collection of data on the student's content deficiencies is most critical because the analysis of these results provides the means for determining where to begin in step two.

Prior to step two, the student's standardized test results are further analyzed in order to form a profile of generalized mathematics deficiencies which is keyed to the Kent State University Mathematics Checklist (1975) in an attempt to bracket these deficiencies with specific content statements. For example, it may be known that the student has some sort of difficulty with addition involving regrouping. The clinician then translates this information into the relative sections of the Checklist, e.g., place value and addition, and selects appropriate entries which elaborate on the general difficulties, e.g., renaming numerals in several different ways, naming the sum of a two-place whole number and a one-place whole number with single regrouping (ones to tens), and naming the sum of a three-place whole number and a two-place whole number with two regroupings. Thus, the purpose at this point is to tentatively identify those elements of the mathematics checklist which will be

used in step two of the model. (It should be noted that the KSU Checklist is a 30 page comprehensive checklist of mathematical concepts for grades K-8.)

Step two represents one of the most critical components of the $(MD)^2$ model for it is at this stage that the specific mathematics deficiencies are isolated. This isolation process occurs through oral interview of the student centering on questions designed to translate checklist entries into specific tasks at the concrete, representative, and abstract levels.

In conjunction with the identification of specific mathematics deficiencies, the clinician identifies the student's "mathematics style." This refers to the observation of the student's utilization of what are referred to as "response modes" and "response formats" in reaction to various "presentation formats."

The $(MD)^2$ model utilizes the following operational definitions for "presentation formats" and "response modes and formats."

Presentation formats can be identified as the following:

Auditory (A): those questions which are posed solely through oral means. The student is asked to respond to that which is heard.

Visual (V): those questions which are posed solely through visual means. The student is asked to respond to that which is seen.

Auditory-Visual (A-V): those questions which are posed through both oral and visual means. The student is asked to respond to that which is heard and seen.

Forced Response: the student must use a given response mode.

Open Response: the student may select a response mode.

Generative: those questions which call for the student to generate the correct response.

Non-generative: those questions which call for the student to select the correct response from a given set of responses.

Response Formats and Modes can be identified as the following:

Oral Concrete (OC): the student responds to a given question by orally describing the situation in terms of concrete objects.

Oral Representative (OR): the student responds to a given question by orally describing the situation in terms of a model or pictorial representation of concrete objects.

Oral Abstract (OAb): the student responds to a given question by orally describing the situation in terms of abstract symbols.

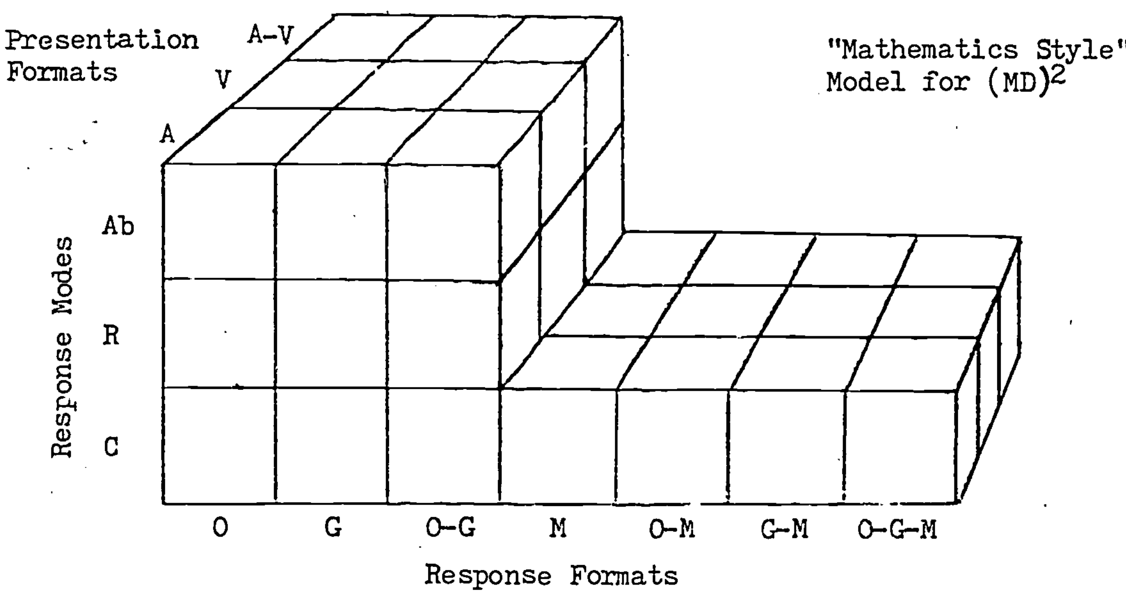
Graphic Concrete (GC): the student responds to a given question by describing, in graphic form, the situation in terms of concrete objects.

Graphic Representative (GR): the student responds to a given question by describing, in graphic form, the situation in terms of a model or pictorial representation of concrete objects.

Graphic Abstract (GAb): the student responds to a given question by describing, in graphic form, the situation in terms of abstract symbols.
Manipulative Concrete (MC): the student responds to a given question by manipulating concrete objects to describe the situation.

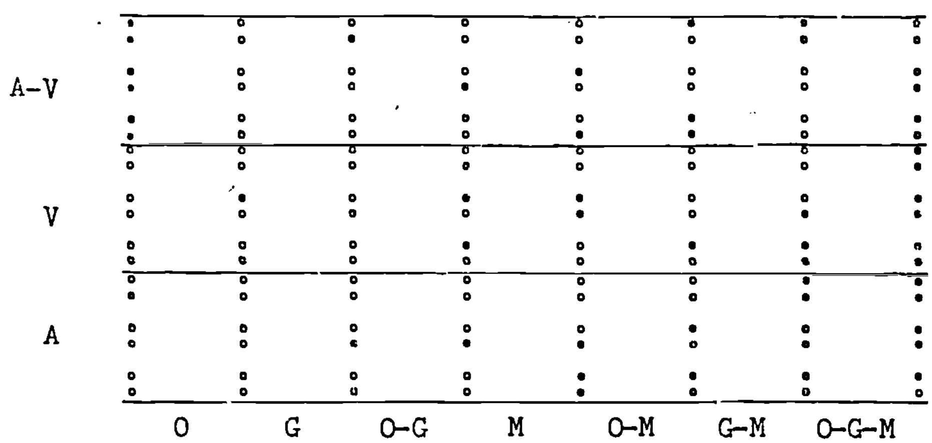
There are several response modes which involve combinations of the aforementioned "unary" response modes. These are:
Oral-Graphic Concrete (O-GC): the student responds to a given question by describing, both orally and in graphic form, the situation in terms of concrete objects.
Oral-Graphic Representative (O-GR): the student responds to a given question by describing, both orally and in graphic form, the situation in terms of a model or pictorial representations of concrete objects.
Oral-Graphic Abstract (O-GAb): the student responds to a given question by describing, both orally and in graphic form, the situation in terms of abstract symbols.
Oral-Manipulative Concrete (O-MC): the student responds to a given question by describing, both orally and by manipulation, the situation in terms of concrete objects.
Graphic-Manipulative Concrete (G-MC): the student responds to a given question by describing, both in graphic form and by manipulation, the situation in terms of concrete objects.
Oral-Graphic-Manipulative Concrete (O-G-MC): the student responds to a given question by describing, in oral and graphic form and by manipulation, the situation in terms of concrete objects.

Graphically, the $(MD)^2$ mathematics style model is shown below.



The previous plate graphically represents the mathematics style component of the $(MD)^2$ model. Note that in a diagnostic session, the trained clinician will use subjective judgment in the selection of which cells to diagnose. For a given checklist entry certain cells are inappropriate and still others can be eliminated on a selective basis. Thus, for a given checklist entry the diagnostician may ask questions based on from one to, say, four cells of the model.

As the content diagnosis progresses the clinician needs to record, in some detail, the events which are (or may be) relevant to identifying the student's deficiencies. The figure below represents one method of recording the presentation format and response mode and format for each major question. It should be noted that the figure has been reduced in size.



Cells are filled with either Ab, R, or C followed by a numeral indicating the order in which the question was asked.

Clearly, the information recorded on this form is invaluable for profiling a student's mathematics style. It does not, however, provide information relative to specific questions asked nor does it make record of any extraneous factors which may affect student response. The Interview Record sheet indicates that the clinician should record the number of the question being asked. Since all of the questions cannot be determined in advance--they will depend on student responses to previous questions--some method of recording questions is needed. For this reason it is strongly suggested that the interview be audio-taped and, if possible, video-taped. This will allow the clinician to reconstruct the session for purposes of further analysis.

When step two of the $(MD)^2$ model is completed the clinician should have a relatively clear picture of the student's mathematical

strengths and weaknesses as well as an understanding of the student's "mathematics style." Consequently, it would be possible to terminate the diagnosis at this point and prescribe corrective teaching based on these findings. However, the $(MD)^2$ model has an additional component which enhances the possibility for successful prescriptions--the diagnosis of general educational cognitive style.

Step three, the educational cognitive style component of the $(MD)^2$ model, is based on an abbreviated version of the model proposed by the American Educational Science Association. The clinician should gather cognitive style data through observation and interview whenever possible, however there is a preference test which can be used either solely or in conjunction with the other two methods.

Following the observation, interview, and preference testing of the individual to gather data on educational cognitive style the clinician is prepared to initiate step four of the $(MD)^2$ model. This step represents the stage during which the data collected in steps one, two, and three is analyzed into a student diagnostic profile. This profile includes data pertaining to: (1) general information reflecting student background, (2) the student's specific mathematics deficiencies and mathematics style, and (3) the student's educational cognitive style. The clinician's task is to piece together this information to form a profile representing the diagnosis of the individual.

One aspect of stage four is the search for consistency between the student's mathematics style and educational cognitive style. A given student may, during the content diagnosis, exhibit a tendency to react positively to questions presented in a visual format but negatively to those with an auditory format. If this same student's educational cognitive style map indicates a minor or negligible T(VL), T(VQ), or Q(V) element then the diagnosis may be incomplete--at the least, it must be reviewed for errors. If, on the other hand, the findings from the mathematics style diagnosis and the findings from the educational cognitive style diagnosis match then the clinician can be reasonably certain that steps two and three of the $(MD)^2$ model were successful.

An additional element of consistency can be checked at this stage of the diagnostic process. It is possible to describe a cognitive style map which indicates the ability to deal with mathematics presented at the concrete, representative, and abstract levels. Using the information provided by such maps the clinician is able to determine whether an individual's inability to successfully deal with mathematics presented at a particular level of abstraction is caused by a deficiency in certain cognitive style components or by a lack of experience with a given level of abstraction. That is to say, if an individual's map indicates the presence of those components

necessary for working at the concrete level but the mathematics style aspect of the content diagnosis suggests that the student has difficulty describing or completing exercises at this level then, perhaps the student has not had sufficient experiences with concrete models. The concept of division serves as an excellent example of the importance of this step of the analysis done by the clinician. Consider a student that can complete the division algorithm but cannot use counters to illustrate the process. Is it because the student is incapable of modeling (because of a lack of cognitive style components) or is it because the student has never had to model the operation and therefore lacks understanding and experience? The question poses interesting problems in designing corrective procedures.

Step five of the $(MD)^2$ model represents the first step in the process of prescription development. Diagnosis based on cognitive style constructs will be of little value if prescribed activities do not provide a high probability of student success. Therefore, the diagnostician must not only diagnose the student, but also those tasks which may be used in subsequent instruction (Hill, 1974). In this manner cognitive style diagnosis not only provides for the identification of the unique structures each individual brings to a learning situation, but it also allows for the translation of this uniqueness into proposed programs of instruction (Radike, 1973).

In step five, the clinician maps the instructional resources--the purpose being to determine the cognitive style conditions of those aspects of the educational environment which may be used for corrective teaching. Included in this category, and therefore subject to mapping, are persons (those individual's that may play a role in the subsequent teaching of the child, e.g., teacher, tutor, librarian, and counselor), processes (those activities which may be used in subsequent teaching, e.g., methods of instruction), and properties (materials used in subsequent teaching, e.g., audio-tapes, films, books, worksheets, and manipulatives). Generally speaking, diagnosis of properties provides data for the matching of symbolic orientations, diagnosis of processes provides data for matching modalities of inference, and diagnosis of persons provides data for matching cultural determinants. It should be noted, however, that these three components of cognitive style must be considered as inseparable and consequently need to be viewed as a totality.

The mapping process at this stage is the same as it was for step three. The clinician must map the instructional resources in the same manner as mapping the student who will come into contact with these resources. That is, the symbolic conditions of elements of the instructional process are determined so that individuals can be matched to these for prescriptive purposes. As an example, assume a possible corrective teaching technique involves having the child work with a peer on the analysis of word problems according to certain

delineated procedures--a fixed step-by-step approach. The clinician mapping such a task may arrive at the following condition of this task.

$$\left\{\begin{matrix} T(VL) \\ T(VQ) \\ Q(V) \\ Q(CET) \end{matrix}\right\} \times \left\{\begin{matrix} A \\ F' \\ \\ \end{matrix}\right\} \times \left\{\begin{matrix} M \\ \\ \\ \end{matrix}\right\}$$

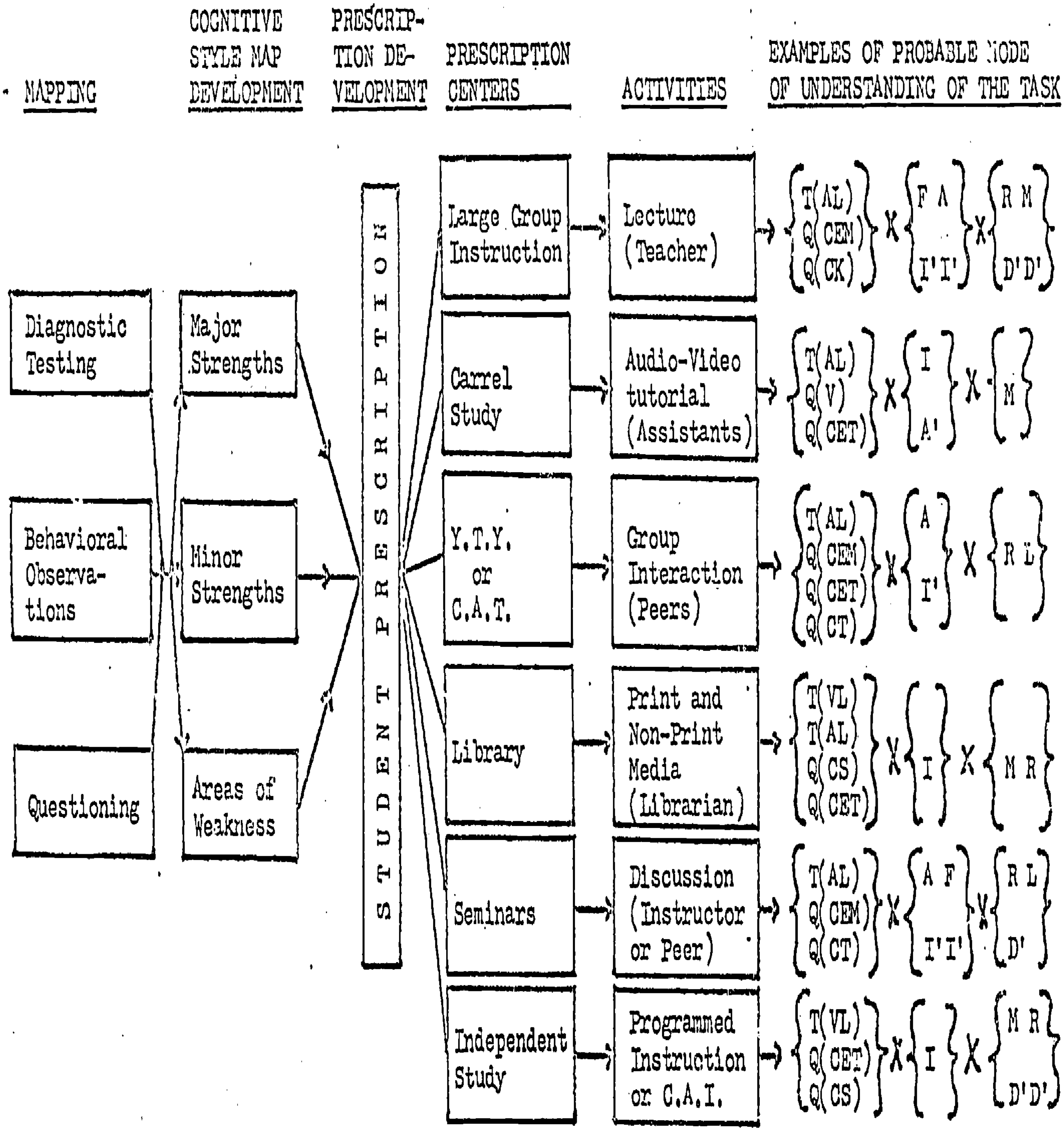
For the child whose map is shown below this may not be a valuable experience.

$$\left\{\begin{matrix} T(AQ) \\ Q(A) \\ Q(T) \end{matrix}\right\} \times \left\{\begin{matrix} I \\ \\ \end{matrix}\right\} \times \left\{\begin{matrix} R \\ \\ \end{matrix}\right\}$$

That is, the interface between the conditions of the task and the cognitive style of the student do not sufficiently match.

It should be noted that, theoretically at least, step five may only have to be completed once. If the clinician, or the teacher, maps all of the resources available then these same maps can be filed for use in the development of prescriptions for several students. Therefore, a goal of the clinician should be the compilation of resource maps for quick reference in future situations.

Step six of the $(MD)^2$ model involves the preparation of prescriptions and, of course, these prescriptions are based on the information gleaned from steps one through five. Cognizance of individual differences is no less critical at this stage than it is at any other stage of the $(MD)^2$ model. The purpose of the prescription is to relate the individual's unique $(MD)^2$ profile to the available resources in a manner which will provide the greatest probability of success. Some possible prescriptive routes based on cognitive style diagnosis of both individuals and educational tasks are presented in the following table.



Personalizing Education (Adapted from Bowman, Birch, Hill, & Nunney, 1974)

The format of the prescription may vary but there are certain characteristics which each prescription should include. One criteria is comprehensiveness. The prescription should include a synopsis of all data obtained in steps one through five organized and presented in a manner which clearly conveys the clinicians analysis and interpretation of the data. Any formal and/or informal tests which were administered during the course of the diagnosis should be described along with student reactions to these tests. Each major component of the $(MD)^2$ model, i.e., content diagnosis, mathematics style, and educational cognitive style, must be discussed in great detail. Each component could be mentioned in isolation, however, because of their interrelatedness, a concurrent discussion designed at weaving the three into one overall picture is preferred. Separate discussions of the components does, however, have the advantage of providing the teacher with a clear-cut picture of each. Also, because of the strong relationship between mathematics style and educational cognitive style it may be desirable to present a cogent discussion of these two components and a separate discussion of specific content deficiencies. Nevertheless, this should still be followed by a synthesis of all three as a hedge against the teacher basing corrective teaching on the content diagnosis alone. The teacher must not overlook the prescriptive benefits gained from the model's ability to diagnose the student's capabilities in the areas of symbolic orientations, modalities of inference, and large group, small group, and independent study.

One final point on prescription deserves--indeed demands--attention. Any diagnosis will indicate to the clinician certain strengths and weaknesses which will necessarily affect the design of corrective teaching procedures--the question is "How"? The clinician has two alternatives to pursue.

On the one hand, a prescription can be written which calls for corrective teaching techniques designed to utilize strengths in both mathematics style and cognitive style to build skill and understanding with those areas identified as content deficiencies, for the most part ignoring style weaknesses in the instructional design. For example, consider the student with the following over-simplified profile.

Content Deficiency
- Renaming fractions in simplest form
- Comparing fractional numbers

Mathematical style
- Auditorily oriented
- Uses concrete objects

Educational Cognitive Style

$$\left\{\begin{matrix} T(AL) & T'(VQ) \\ Q(A) & \\ Q(CET) & \end{matrix}\right\} \quad X \quad \left\{\begin{matrix} I \\ \\ \\ \end{matrix}\right\} \quad X \quad \left\{\begin{matrix} M \\ \\ \\ \end{matrix}\right\}$$

Under the above philosophy of corrective teaching this student may be asked to sit alone at a listening station with audio tapes keyed to cards with appropriate numerals on them. These tapes describe the process of renaming fractions in simplest form by way of using concrete objects and directs the child to model fractions using cuisenaire rods placed at the station. Little or no reference would be made to visually-oriented resources, e.g., filmstrips, nor would any concerted effort be made to have the student work with peers or watch the teacher model some examples at the board.

The other alternative position on corrective teaching is to utilize both strengths and weaknesses in mathematics style and cognitive style in order to eliminate content deficiencies. For the example just given the prescription would include such activities as peer assistance, visual aids, teacher demonstration, etc. The rationale for using style weaknesses is that through use they may develop into strengths.

On the surface it seems as though the second alternative would be most beneficial. In fact, in most instances it would be the most probable route for eliminating deficiencies. After all, it does provide the student with a greater variety of opportunities to identify errors and misconceptions. However, this does not mean that this philosophy of corrective teaching will work best for all. Those students with serious content deficiencies may become even more confused by having to deal with two deficiencies at once, namely, content deficiencies and style deficiencies.

To summarize, prescriptions calling for corrective teaching demand careful consideration. At the risk of over-simplifying, students with major content deficiencies should receive corrective instruction designed to utilize their style strengths to alleviate these deficiencies, while students with minor content deficiencies should receive corrective instruction designed to utilize their style strengths and weaknesses to alleviate their deficiencies. The determination of what are major or minor content deficiencies should be based on analysis of test results and the subjective judgment of a trained clinician.

Step seven of the $(MD)^2$ model, implementation, refers to the means of incorporating the results of the entire diagnostic process into the instructional program. This is not to be confused with the act of corrective teaching which is not a part of the model since it is an activity carried out by the classroom teacher. The implementation of the $(MD)^2$ model refers to the manner in which the diagnosis is transmitted to the teacher. The vehicle for accomplishing this step is the clinician-teacher conference. The prescription report discussed in step six may simply be delivered to the teacher for consideration, however, it is strongly suggested

that a conference be arranged so that the possibilities of misinterpretation are lessened prior to the initiation of corrective teaching procedures. When both parties have come to a consensus concerning the findings of the diagnosis and the purpose and rationale of the prescription the teacher may then determine the manner in which instruction will take place.

Evaluation, step eight, is based primarily on teacher-input after corrective teaching has begun, once again through clinician-teacher conferences. The teacher input should be founded on observations and test results. An additional aspect of the evaluation step is observation, by the clinician, of the child at work in the classroom. Observation by both the clinician and the teacher is designed to allow for a "comparing of notes" (which may assist the development of greater interrater reliability for later referals) and to form a common base of knowledge concerning the student's status to insure the success of the clinician-teacher conference.

Feedback into the diagnostic system, step nine, may be the result of step eight, the clinician-teacher conference. The re-entry step will vary for individual students. For some it may be necessary to begin the entire diagnostic process anew. For others, re-entry may take place at either step two (content diagnosis incorporating mathematics style), step three (educational cognitive style diagnosis), step four (analysis of steps one, two, or three), or step six (prescription writing).

Thus, the diagnostic process has traveled full circle. If carried out properly there should be a wealth of information available concerning not only what the student does and does not know but also concerning the manner in which the student does and does not take on meaning. The real value, however, is not in simply having this information but in using it. A carefully planned follow-up program of corrective teaching is critical to the success of any diagnostic venture.

Conclusion and Recommendations for Further Study

By way of conclusion it seems appropriate to review some of the more pertinent aspects of the model. First, it is essential that the reader understand the clinical nature of the model. It is designed to describe a possible diagnostic process which can be carried out by a trained clinician--it may have some value in classroom diagnosis but this is not the intended target. Second, it is important to note that this model is not intended to be used only with those students that have severe mathematics difficulties. The mathematics style and educational cognitive style components of the model make this model valuable for the diagnosis of any

student of mathematics. Third, in the actual diagnosis of an individual's mathematics style it is not intended that all thirty-nine cells be tested. It is necessary for the clinician to use subjective judgment and select those cells which are most appropriate for the task at hand. Fourth, the $(MD)^2$ model, when implemented, does not describe a diagnostic test from which one teaches. There must be intermediate steps between the administration of this model and the actual instructional process.

The Model for Diagnosing Mathematical Deficiencies is presented as one conceptualization of the manner in which diagnosis could proceed. The evidence on interrelationships among the abundance of factors affecting development suggests that each student brings to each learning situation a differential combination of unique capabilities and abilities, each at a particular stage of development. Diagnosis, then, should strive to describe these capabilities and abilities and the factors which affect them. Its ultimate purpose is to facilitate the construction of individualized prescriptions for uniquely organized persons. The $(MD)^2$ model represents one attempt to reach this goal.

The purpose of this paper was to develop and describe a clinical model for diagnosing mathematical deficiencies which incorporates cognitive, affective, and psychomotor aspects of educational cognitive style. This model is designed in such a way as to reflect consistency with the view that the task of diagnosis is to describe a personality as well as a person's subject matter deficiencies.

It is important to note that the purpose of this paper was not to describe processes through which the diagnostic model could be implemented. Indeed, application concerns are not (nor should they be), factors in the design stage of model development. Hypothesizing on <u>possible</u> application difficulties prior to the development of a model may cause undue restrictions and limitations to form in the mind of the designer. Comments relevant to this point can be found throughout the literature. For example,

> A basic innovative design may well be 'useless' in the sense that it has little or no application immediately to schools as educational institutions.... Concern for the immediate applicability of the findings can distract the researcher, narrow his efforts and hasten him to unjustified conclusions. (Brickell, 1961, p. 82)

Thus, Brickell described three distinguishable phases of innovation: design, evaluation, and demonstration; and their ideal settings which are, respectively: freedom, control, and normality.

Werner (1968), in commenting on Brickell's notion of three phases of innovation, stated

> Design efforts cannot be conducted in evaluation settings because experimental controls of the type needed for adequate evaluation are restrictive by their very nature. These restrictions reduce the freedom to explore for something better. The ordinary, unenriched setting needed for the demonstration of a proven innovation is the setting least likely to generate new designs. The observer of a demonstration needs to see the demonstration of the innovation as part of the normal, ongoing program in a school like his own. For these reasons, therefore, the circumstances needed for the design of an innovation cannot be reconciled with those needed for proper evaluation and demonstration of the innovation. (Werner, 1968, pp. 89-90)

A similar concern was expressed by Wilson (1973) in a paper calling for more efforts in generative research--specifically clinical intervention research. Citing the unfortunate tendency to evaluate the results of generative research on the basis of criteria designed for confirmative experimental research, Wilson noted that "clinical intervention is either not recognized as research, or is depreciated as research, and in some circles even deprecated as research" (Wilson, 1973, pp. 1-2).

As previously mentioned, Engelhardt (1974) has commented on the difficulties encountered in experimental research without the benefit of a theoretical model. Thus the significance of this paper lies in its purpose, that is, it is the development of a theoretical model, and a description of its accompanying instrumentation, for clinically diagnosing mathematical deficiencies and its relation to the teaching and to the learning of mathematics.

Thus, at the risk of "depreciation" or "deprecation," the present paper is best described as a generative effort to describe a diagnostic model for clinical use in identifying an individual's unique mathematics deficiencies. It should be recognized as a first attempt which has, at this point, only been administered on a limited basis. Now, and only now, it needs to be examined, and possibly adapted, for use in diagnostic-prescriptive settings.

Because this study is not based on statistical analysis conclusions similar to those found in experimentally-oriented studies cannot be stated. Consequently, this section is devoid of any attempt to state inferences but instead focuses on areas of further study and needed research.

The design process used in the development of the $(MD)^2$ model suggests the following areas of needed research and development.

1. The $(MD)^2$ model suggests a hierarchical checklist for use in determining a student's content deficiencies. This hierarchy is based on expert judgment but perhaps other means of hierarchy validation such as Guttman analysis should be attempted.

2. The $(MD)^2$ model suggests the use of a standardized diagnostic test to obtain entry level information on the child's content deficiencies. Does a standardized diagnostic test provide sufficient data for transferring general difficulty areas into the checklist?

3. The $(MD)^2$ model suggests instrumentation to be used in conjunction with a checklist for diagnosing a student's mathematical style. A next step would be the development of a battery of items for each entry in the checklist.

   Several questions would need attention:

   (a) Should the development of this battery begin with one or two concept clusters or should the entire checklist be subject to item development? (This question becomes critical when one realizes that the checklist is not sequenced across concepts.)

   (b) How many items are needed for each entry of the checklist? (It is necessary to consider the possible presentation and response formats for an individual entry before this question can be answered.)

   (c) Concerning the internal structure of the item battery, should the questions be open or forced response? Generative or non-generative response? Should there be a mixture of these response types?

Considering implemention of the $(MD)^2$ model in diagnostic prescriptive settings, the following areas need to be inspected.

1. The $(MD)^2$ model, while not so complicated that it can't be implemented, does require a certain amount of expertise. What procedures must be developed for training mathematics clinicians in the use of the $(MD)^2$ model? For training classroom teachers in its use? In what ways can computer capabilities be used to simplify the data collection and record keeping aspects of the $(MD)^2$ model?

2. The $(MD)^2$ model is designed for use in a mathematics clinic, or at least in a program with clinical procedures. Several authors have suggested that the adaptation of clinical practices to classroom techniques is beneficial (Buswell, 1935; Callahan, 1973; Denmark, 1974). What adaptation is necessary before $(MD)^2$ can be used in classroom diagnosis?

3. The educational cognitive style component of the $(MD)^2$ model has been successfully used at the upper elementary, secondary, and college levels. The $(MD)^2$ model itself is designed for use at any level of mathematics instruction, however the vehicle used to describe the model is elementary school mathematics. What changes, if any, are needed in the model design before it can be implemented at the secondary and post-secondary levels?

4. It has been suggested that longitudinal study is needed to determine the usefulness of cognitive style diagnosis (Sternberg, 1975). Thus, a long-range testing program may need to be established before the value of a model such as $(MD)^2$ can be fully evaluated.

Assuming that implementation attempts are successful, certain questions on the actual use of the $(MD)^2$ model need to be addressed.

These include:

1. The $(MD)^2$ model is designed to provide an individual profile of three major areas: content deficiencies, mathematics style, and educational cognitive style. Is there benefit to be gained by fractionating the model and using only one of the components? Any two of the components?

2. The $(MD)^2$ diagnostic model assumes that, among other methods, a dyadic interview will be used to gather data. If a student does not perform appropriately on a diagnostic instrument it may be due to other factors aside from the student not understanding the concept being assessed. What critical factors can be identified that affect clinician-student interactions? What role does clinician cognitive style play in determining the success of the interview? Is there a "most effective" clinician cognitive style?

3. The $(MD)^2$ model is designed to describe individuals from a variety of perspectives. What are the individual difference variables which might affect performance in the diagnostic setting that have not been considered? How does use of the model affect children? Are cognitive styles of individuals content specific?

4. The $(MD)^2$ model provides the opportunity to use diagnosed strengths and weaknesses in a variety of ways. Does remediation based on using cognitive style strengths to eliminate content deficiencies prove to be more successful than using a combination which stresses strengths but also attempts to eliminate weaknesses through use?

The above represent questions generated from the development of the Clinical Model for Diagnosing Mathematical Deficiencies--$(MD)^2$. Since this paper is concerned with only the design stage of model development it is clear that a significant amount of work must be done before implementation. This work now begins.

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